A simple alternative model for corporate investment
Elliot Aurissergues

To cite this version:

HAL Id: hal-01558216
https://hal.archives-ouvertes.fr/hal-01558216
Submitted on 7 Jul 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
A simple alternative model for corporate investment

Elliot Aurissergues

Draft JUNE 2017
Abstract

The aim of this paper is to provide a simple model in which fall in interest rate fails to increase corporate investment through the traditionnal user cost of capital channel. The motivation is the lack of empirical evidence for this channel and the ineffectiveness of the loose monetary policy with regard to corporate investment. I develop a simple model of adverse selection. It allows to microfound a linear relation between corporate investment and cash flow which ”kills” the user cost channel. I solve the model analytically and find that lower interest rates decrease corporate investment.

JEL Classification: D92,E32,E43,E44,G30

Keyword: corporate investment, real interest rate, user cost of capital, financial frictions, adverse selection, precautionnary saving

Elliot Aurissergues

PHD student, Paris School of Economics and University Paris Pantheon Sorbonne

elliot.aurissergues@gmail.com
Introduction

According to the standard theory of investment, low real interest rates lower the user cost of capital and triggers an increase in desired capital stock and thus in corporate investment. The past decade provides an imperfect but interesting test for this theory. Between 2008 and 2016, the federal fund rate was stuck at the zero lower bound. Inflation remained positive, so short term real interest rate were negative for nearly a decade. The rise did not happen. Net corporate investment in the US was negative from 2008 to 2010 and recovered very slowly after. After 2012, it remained lower than during previous historical recoveries in which real interest rates were much higher.

This inability to stimulate investment is striking and questions the standard theory. It can be argued that the financial crisis and the following slow recovery are not a particularly good natural experiment. "Big" rise in uncertainty, increased market power and a depressed aggregate demand can explain a depressed investment and thus a low interest rate. But, two other unrelated piece of evidences should cast serious doubt about the neoclassical theory.

First, when estimating the effects of a "shock" on real interest rates, economists do not find large effects on corporate investment. In Bernanke and Gertler (1995), the response of corporate investment to monetary policy shocks is small and very delayed. The response is not significant before eight months and the peak intervenes...twenty months after the shock. The contrast with residential investment, whose response is quick and large, is striking. Given this difference of timing, it would not be implausible that corporate investment does not react at all to monetary policy shocks but reacts to the response of residential investment through old keynesian investment accelerator for example. This small response in VAR models is also a serious problem for the neoclassical theory. If well identified, monetary policy shocks are not correlated with uncertainty, depressed aggregate demand etc. There is still a lot of controversy about identification of monetary shocks but this finding of Bernanke and Gertler have never been challenged at our knowledge.

A second strand of the literature have challenged the neoclassical theory. Estimation of investment equation struggle to find a significant effect of real interest rates. The consensus in the literature is that estimating short run elasticity of investment to interest rate with aggregate datas do not provide any evidence backing a significant effect of interest rate on business investment (Blanchard 1986, Caballero 1994, Bernanke and Gertler 1994, Chirinko 1993, Sharpe and Suarez 2015). When measuring the different channels of monetary policy, Bernanke and Gertler (1994) shows that the response of business investment to a recessionnary FED fund rate shock is negative but small and lagging behind the large response of residential investment and nondurable consumption. The lag and the size of the response actually suggest more a side effect of the residential investment response through accelerator phenomena than
a user cost effect. Estimates of long run elasticity (Caballero 1994, Schaller 2006) and studies using microeconomic datas (Cummins, Hassett and Hubbard 1994, Chirinko Fazzari and Meyer 1998, Molin, Smets and Vermeulen 2001) provide better evidences for a user cost of capital effect on investment. However, it is still unclear if their findings imply a high elasticity of investment to interest rate as noted by Sharpe and Suarez. For example, the estimate of user cost elasticity in Cummins, Hassett and Hubbard (1994) is unchanged when real interest rate is replaced by a fixed discount rate in the measure of the user cost (Sharpe and Suarez 2015). Direct measure of investment sensitivity to interest rate are not better. For example Kothari and Warner(2015) finds that interest rate is unable to predict corporate investment whereas for example interest rate predicts noncorporate investment. Facing these very mixed empirical results, Sharpe and Suarez(2015) have proposed a completely different approach. Instead of using econometric techniques to identify correlation, they directly ask to CFO (Chief Financial Officers) in what extent their investment decision is sensitive to interest rate. Results are very instructive. 68 % of CFO says that their investment plans will remain unchanged if interest rate falls.

Alone, a bad post financial crisis performance despite historically low real rates, small response to monetary policy shocks in VAR, small effects of real rates in investment equation estimation with macroeconomic datas and microeconomic datas, and a majority of firms claiming they do not change their investment plan following a fall in interest rates, would not be necessarily conclusive. Together, they suggest to look for other theories in which corporate investment would not react to real interest rate through the user cost channel.

This paper contributes to fill this gap. I provide a simple model of adverse selection on capital markets. I find that in infinite horizon, the Incentive Compatibility Constraint takes the form of a linear relation between investment and firm cash flows. The user cost of capital is no longer relevant. I embed this relation in a dynamic macroeconomic model and study the response of aggregate investment to permanent and temporary changes to real rates. I specify the adverse selection problem in the first section, define and characterize the equilibrium in the second, and solve the macroeconomic model in teh third.
1 The adverse selection problem

In this section, I outline a partial equilibrium model of capital markets in which firms try to get funds from lenders in order to finance their investment. Firms hold private information about their riskiness. Firms try to signal their riskiness by constraining the amount they borrow to be a fraction of their retained earnings.

1.1 Firms

Production function  The economy is populated by a continuum of firms. A firm $j$ have a production function

$$Y^j = \pi K^j$$ \hspace{1cm} (1)

Two type of firms  There are two type of firms in the economy: Bad firms denoted with superscripts $B$ and good firms denoted by the superscript $G$. Bad and good firms does not differ by their productivity but by their riskiness. At each period, a good firms has a probability $\kappa$ to become a bad firm. With probability $1 - \kappa$, it remains a good firm. A bad firm still produce $\pi K^j$ but has a probability $1 - \lambda$ to exit at the next period. When it exits, a firm produces nothing and its capital stock is worthless. Neither the lender nor the borrower recover anything. I summarize the timing by the following tree.

The firm problem  Shareholders wants that firms maximize the discounted sum of their expected dividends.
\[ V(K, B) = \max_{K', B', I, S, E, d} d + \frac{1}{1 + r} EV'(K', B') \]  

(2a)

w.r.t \( K' = K + I \)  
(2b)

\( B' = B + r^E E \)  
(2c)

\( \pi K - B = d + S \)  
(2d)

\( I = E + S \)  
(2e)

\( (\pi K - B) \geq d \geq (1 - s)(\pi K - B) \)  
(2f)

\( 0 < E < E^{\text{max}} \)  
(2g)

The first constraint is the law of motion for capital stock. Next period capital stock is equal to previous one plus investment. There is no depreciation. The second constraint is the law of motion for interest repayments. Next period interest repayments \( B' \) are equal to current one plus the new borrowing \( E \) times the specific interest rate charged on the firm \( r^E \). Indeed, I assume that firms finance themselves by borrowing at infinite maturity. I explain the equation with more details in a dedicated paragraph below. The equation (2C) split the income of the firm \( \pi K - B \), equal to the production minus interests repayments, between dividends denoted \( d \) and retained earnings denoted \( S \). The investment \( I \) is financed by retained earnings \( S \) and borrowing \( E \). Dividends are constrained to have a minimal and a maximal value. They cannot be higher than the income of the firm and should be bigger than \( (1 - s)(\pi K - B) \) where \( s \) is an exogenous parameter between 0 and 1. Borrowing should also be set between 0 and a maximal positive value \( E^{\text{max}} \).

**Loans** I assume that loans have infinite maturity. Interest rates on past loans are fixed. Only interest rate on new loans may vary. Thus for an amount \( E_0 \) borrowed at period 0 , the firm should pay the lender \( r_0^E E_0 \) at each period. At a given period \( t \), the total repayment \( b_t \) of the firm is the sum

\[ b_t = r_{t-1}^E E_{t-1} + r_{t-2}^E E_{t-2} + r_{t-3}^E E_{t-3} + \ldots + r_{t-n}^E E_{t-n} + \ldots \]

where \( E_{t-n} \) is the amount of money borrowed at period \( t - n \) and \( r_{t-n}^E \) is the interest rate at period \( t - n \) which includes a firm specific risk premium.

This assumption allows me to derive a very simple credit constraint. Allowing for shorter maturity are interesting but introduces complex issues about optimal maturity design which is not the core of this paper. However, it is worth noting it is better for firms to accumulate short term assets and long term liabilities in this framework.
Solving the firm problem  The firm problem is not always well defined. The discounted sum of dividends does not always converge because dividends may grow faster than the safe interest rate for some firms. When this is the case, I rely on heuristic arguments to determine the best strategy of the firm.

1.2 Lenders
There is a continuum of lenders which behaves in a competitive way. They have the choice between lending to firms and buying a one period bond generating a safe return \( r \) which is the safe short term interest rate. They can observe earnings, dividends, investment and savings of a firm but they do not observe the true type of the firm. Firms will try to reveal their type by sending an appropriate signal to lenders. As I will show, the signal will be the ratio of investment over retained earnings \( \frac{I}{S} \).

Before defining and characterizing the equilibrium, I introduce some notations and compute interest rate for each type of firms.

1.3 Interest rates and values
Suppose that good and bad firms have fully revealed their type to lenders. Lenders should be indifferent between lending one unit of good to a good firm (resp. bad firm) and receiving \( r_G \) (resp. \( r^B \)) unit of goods at each period while the firm survive and buying a safe assets generating a safe return \( r \) at the following period.

\[
\begin{align*}
    r^B \lambda Q^B &= 1 + r \\
    r^G \left[ (1 - \kappa)Q^G + \kappa Q^B \right] &= 1 + r
\end{align*}
\]

(3a)  \quad (3b)

Where

\[
\begin{align*}
    Q^B &= 1 + \frac{\lambda}{1 + r} Q'^B \\
    Q^G &= 1 + \frac{1}{1 + r} \left[ (1 - \kappa)Q'^G + \kappa Q'^B \right]
\end{align*}
\]

(4a)  \quad (4b)

\( Q^B \) (resp. \( Q^G \)) is the discounted sum of one unit of goods delivered at each period by a bad (resp. good) firm while the firm survives.

1.4 Values and policy rule definition
I denote \( V^B \) and \( V^G \) the values of respectively good and bad firms. I denote \( (I^G, S^G) \) the policy rule of a good firm for retained earnings and investment. Other choice variables can be deduced
using equations (2b)-(2e). Similarly, \((I^B, S^B)\) is the policy rule of bad firms. Both values and policy rules are not real numbers function of the state variables: capital stock \(K\) and debt \(B\).

The value of bad firms will depend to the signal it will send to lenders. If it sends the signal which allows lenders to identify it as a bad firm, it will have to pay the interest rate charged on bad firms \(r^B\). If it sends the same signal as good firms, it will pay the interest rate \(r^G\). I define the value of the firm for both case.

Computing the respective value associated to these two strategies raises some issue. In finite horizon, it is possible to start from the last period and to compute what is the best strategy in the last period. Then, you can compute the value of the two strategies for the previous period and repeat the same process until the current period. Such a solution is not available in infinite horizon. I use the following critical assumption

**Hypothesis 1** In period \(t\), a bad firm will compute its value using the assumption it will send the ”bad” signal in period \(t + 1\)

With this assumption, I can define the value of sending the ”bad” signal

\[
V^{B,B} = d + \lambda \frac{1}{1+r} V^{rB,B}
\]

\(w.r.t\ K' = K + I\)

\(B' = B + r^B E\)

\(\pi K - B = d + S\)

\(I = E + S\)

and the value of sending the ”good” signal in period \(t\) is

\[
V^{B,G} = d + \lambda \frac{1}{1+r} V^{rB,B}
\]

\(w.r.t\ K' = K + I\)

\(B' = B + r^G E\)

\(\pi K - B = d + S\)

\(I = E + S\)

## 2 Equilibrium

### 2.1 Equilibrium definition

I can now define the separatin equilibrium of this economy
Definition 1 An equilibrium is a vector \((r^B, r^G)\), a vector of policy function \((I^B, S^B)\) and a vector of policy function \((I^G, S^G)\) such that

\[
\begin{align*}
&\forall K, B, V^{B,B}(K, B) \geq V^{B,G}(K, B) \\
&\text{• } (I^G, S^G) \text{ are the optimal policy function for the program (2) with } r^E = r^G. \\
&\text{• } (I^B, S^B) \text{ are the optimal policy function for the program (2) with } r^E = r^B.
\end{align*}
\]

The first condition states that all bad firms have an incentive to send the "bad" signal. The two others states that both good and bad firms adopts the best policy.

2.2 Equilibrium characterization

I can characterize the equilibrium in a very simple way with the following assumption

Hypothesis 2 Parameters \(\kappa, \lambda, \pi\) and the sequence of interest rates \((r_t)_{t \in \mathbb{N}}\) are set such that \(\forall t, r^B_t \geq \pi \geq r^G_t\)

With this assumption, I am able to determine and characterize the best policy of the two type of firms when their true type is revealed to lenders.

Proposition 1 If hypothesis 1 holds, under separating equilibrium, bad firms do not save, invest and borrow. \(S^B = 0, I^B = 0, d^B = \pi K - B\)

Proposition 2 If hypothesis 1 holds, under separating equilibrium, good firms wants to invest and borrows as much as possible. They also distribute the minimum level of dividends \(I^G = s(\pi K - B) + E^{max}, E^G = E^{max}, d^G = (1 - s)(\pi K - B)\)

These propositions are quite intuitive. If bad firms invest and borrow \(I^B_t\), they will receive \(\pi I^B_t\) at each period and will pay \(r^B_t I^B_t\). Thus an investment generates a negative stream of income. If the firm do not borrow, they have to choose between investing \(I^B_t\) generating \(\pi\) at each period with dying probability \(\lambda\) and buying a safe asset. It is possible to show that it is equivalent to the case of borrowing. In a similar way, if good firms invest and borrow \(I^G_t\), they will receive \(\pi I^G_t\) at each period and will pay \(r^G_t I^G_t\). A unit of additional investment integrally financed by debt always generates a positive stream of income. Good firms want to invest and borrow as much as possible.
2.3 The incentive compatibility constraint

For the bad firm, the value of distributing all its profit as dividend and not investing whereas paying a high interest rate on its debt should be superior to the value of paying a lower interest rate whereas investing the same fraction of its earnings than good firms. The incentive compatibility constraint can be expressed in a simple way

**Proposition 3** We have $V^{B,B} \geq V^{B,G}$ if and only if

$$I^G \leq \frac{r^B - r^G}{\pi - r^G} S^G = \psi S^G$$

**Proposition 4** Under assumption (2) Good firms wants to invest and borrows as much as possible, thus the incentive compatibility constraint is binding

$$I^G = \frac{r^B - r^G}{\pi - r^G} S^G = \psi S^G \quad (5)$$

I now detail the computations which gives this expression. A linear value function can be derived for $V^{B,B}$ and $V^B_t$. The solution method is straightforward. I guess that the value is a linear function of capital stock and interest repayments.

$$V^{B,B}(K, B) = \phi_K K - \phi_B B \quad (6)$$

Using undetermined coefficients method, I solve for $\phi_K$ and $\phi_B$.

The value function can be rewritten in the following way.

$$V^t_{B,B} = \phi_K K - \phi_B B = \pi K - B + \frac{1}{1 + r} [\phi'_K K - \phi'_B B]$$

I deduce

$$\phi_K = \pi + \frac{1}{1 + r} \phi'_K = \pi Q^B$$

$$\phi_B = 1 + \frac{1}{1 + r} \phi'_B = Q^B$$

The incentive compatibility constraint implies

$$V^{B,B} \geq V^{B,G} \quad (7)$$

I can now rewrite it

$$\pi K - B + \lambda \frac{1}{1 + r} [Q^B \pi K - Q^B B] \geq \pi K - B - S^G + \lambda \frac{1}{1 + r} [Q^B (K + I^G) - Q^B (B + r^G (I^G - S^G))]$$

By simplifying, I get

$$\frac{1 + r}{\lambda} S^G \geq + [Q^B I^G - Q^B r^G (I^G - S^G)]$$
Using equation (3a) which gives interest on bad firms with respect to safe interest rate, I get the ICC under the compact form highlighted in proposition (3)

\[ I^G \leq \frac{r^B - r^G}{\pi - r^G} S^G \]

### 2.4 Leverage and interest rate

Equation (5) gives a linear relation between investment and cash flows or retained earnings. The ratio \( \frac{r^B - r^G}{\pi - r^G} \) is the leverage on cash flows and I denote it \( \psi \). It is interesting to note that this constraint is about flows and not stocks unlike most of popular financial frictions in macroeconomics. This equation makes the user cost irrelevant to determine investment. However, the leverage is not independent from interest rate. Interest rate affects it through \( r^B \) and \( r^G \). But, interestingly it is an increasing function of the interest rate.

I compute the derivative of the leverage with respect to \( r \)

\[ \frac{\partial \psi}{\partial r} = \frac{(\frac{\partial r^B}{\partial r} - \frac{\partial r^G}{\partial r})(\pi - r^G) + (r^B - r^G)\frac{\partial r^G}{\partial r}}{(\pi - r^g)^2} \]

In first approximation, a fall in \( r \) has the same impact on \( r^G \) and \( r^B \), thus an approximation for the derivative is

\[ \frac{\partial \psi}{\partial r} = \psi(r) \frac{\partial r^G}{\partial r} \]

\( r^G \) is an increasing function of \( r \) and its derivative is positive. Thus, \( \frac{\partial \psi}{\partial r} > 0 \). This result implies that firms will tighten their investment policy for a given cash flow when real interest rates falls. This counterintuitive result is in line with the seminal paper of Stiglitz and Weiss (1977). The intuition is simple. Suppose the owner of a bad firm sends the "good" signal and invest for example \( \psi \) unit of good and borrows \( \psi - 1 \) unit of good. This strategy has a benefit and a cost. The benefit is the following. The \( \psi - 1 \) units borrowed generate a return \( \pi \) and only cost \( r^G \) at each period. The cost is that it should retain one unit of goods into the firm instead of distributing them as dividends and investing them in the safe asset. Suppose now that the interest rate falls. Buying the safe asset generates a lower return and thus the opportunity cost of investing one unit of good into the firm is lower. The cost of sending the "good" signal falls. In the same time, the benefit increases. \( r^G \) is lower. The \( \psi - 1 \) unit of goods borrowed generates more profits at each period. The owner of a bad firm has much more incentive to send the "good" signal instead of the bad one. Owner of good firms have to tighten their investment policy to signal they are good. By lowering the leverage \( \psi \), they will lower the benefit of sending the good signal for bad firms.
3 The macroeconomic model

3.1 The model

From the viewpoint of the economist, the investment equation obtained in the previous section has three appealing features:

First, it is a microfounded relation. There is a clear link between the cash flow leverage and real interest rate.

Second, it is a tractable formula for investment. Adverse selection in infinite horizon is not a simple problem in first thought. Finding such a simple equation is interesting.

Third, a relation between cash flows and investment is empirically plausible. The sensitivity of investment to cash flows is a regular feature of empirical studies (unlike Tobin’s Q or user cost of capital). Finding a simple theoretical justification for it is also quite interesting.

Given this attractive features, it is logical to develop the framework of the previous section in a dynamic macroeconomic model. This model have three blocks: Prices equation, dynamic equations for good firms, dynamic equations for bad firms. Price equations gives interest rates for bad and good firms and the value of the leverage.

\[ Q_t^B = 1 + \frac{1}{1 + r_t} \lambda (1 - \phi) Q_{t+1}^B \]  
\[ Q_t^G = 1 + \frac{1}{1 + r_t} [(1 - \kappa)(1 - \phi)Q_{t+1}^H + \kappa (1 - \phi)Q_{t+1}^B] \]  
\[ r_t^B \lambda Q_{t+1}^B = 1 + r_t \]  
\[ r_t^G \left[ (1 - \kappa)Q_{t+1}^G + \kappa Q_{t+1}^B \right] = 1 + r_t \]  
\[ \psi_t = \frac{r_t^B - r_t^G}{\pi - r_t^G} \]

Equations (10 a) to (10 d) allows to derive interest rates specific to good and bad firms as explained in the previous section. Equation (10 E) gives the leverage of period \( t \) with respect to these two interest rates and the productivity of capital \( \pi \).

Dynamic equations for good firms are

\[ K_{t+1}^G = (1 - \kappa)(K_t^G + I_t^G) \]  
\[ B_{t+1}^G = (1 - \kappa) \left( B_t^G + r_t^G (I_t^G - S_t^G) \right) \]  
\[ \pi K_t^G - b_t^G = S_t^G + d_t^G \]  
\[ d_t^G = (1 - s)(\pi K_t^G - B_t^G) \]  
\[ I_t^G = \psi_t S_t^G \]
Equation (11 A) gives the capital accumulation of good firms at the aggregate level. Next period capital stock is the current capital stock plus investment times the "survival" rate of period t good firms at period t + 1. Equation (11 B) gives the debt accumulation equation for good firms, debt being measured by interests repayments. Equation (11 C) is an accounting equation. Aggregate income of good firms is split between aggregate savings and aggregate dividends. All variables are in period t, so the survival rate is not relevant here. This is also the case for the two others equation which gives the aggregate policy rule of good firms for dividends and investment.

Similar equation can be given for bad firms

\[ K_{t+1}^B = \kappa K_t^G + (1 - \lambda)K_t^B \]  
\[ B_{t+1}^B = (1 - \lambda)B_t^B + \kappa [B_t^G + r_t^G (I_t^G - S_t^G)] \]  
\[ d_t^B = \pi_t K_t^B - b_t^B \]

Bad firms do not invest but a fraction \( \kappa \) of good firms becomes bad at period t and "feed" capital stock and debt of the bad firm sector. All the income of bad firms is distributed as dividends.

The real interest rate \( r \) is exogenous in the model. Indeed, I focus on the response of investment to an exogenous change of \( r \). The exogeneity of the interest rate also allows to solve the three blocks separately

**Proposition 5** The system of equations (8) can be solved independently from system (9) and (10)

Indeed, none of the eight variables of the two systems appears in the five equations of the system (8).

**Proposition 6** The system (9) can be solved independently from system (10)

Because bad firms always remain bad and does not affect interest rates, the capital stock, the debt and the dividends of bad firms do not affect good firms.

**Dynamic of investment** I now show that the system (9) can be reduced to one recursive equation for debt from which it is possible to deduce the dynamic of investment and growth.

The system has to be stationarized. Indeed, the AK production function allows for endogenous growth. the solution is to divide all variables by \( K_t^G \) and to replace \( K_{t+1}^G \) by \( 1 + g_{t+1} \) where \( g \) is the growth rate. Stationarized variables are denoted in lowercase.

The five stationarized equations can be simplified in a system of three equations whose variables are debt \( b \), investment \( i \) and growth
Replacing (11 C) in (11 B) and (11 B) in (11 A) leads to recursive equation for \( b 
\)

\[
\begin{align*}
(1 + g_{t+1})b_{t+1} &= (1 - \kappa)(b_t + r^G_t(\psi_t - 1)s(\pi - b_t)) \\
1 + g_{t+1} &= (1 - \kappa)(1 + i_t) \\
i_t &= \psi_t s(\pi - b_t)
\end{align*}
\] (11a) (11b) (11c)

The equation is nonlinear. To give some intuition, I plot a typical graph associated to the recursive equation in figure (1)

![Graph](image)

**Steady state** I solve analytically for the steady state. The two stationnary debt levels is given by the roots of the recursive equation

**Proposition 7** Equation (13) have two roots \( b^* \) and \( b^{**} \)

\[
\begin{align*}
b^* &= \pi \\
b^{**} &= \frac{r^G(\psi - 1)}{\psi}
\end{align*}
\]
It is easy to see that one of the roots is equal to $\pi$. In that case, all the firms income is distributed as dividends, there is no growth and no investment, so interest repayments over capital stock remains at a fixed level. This equilibrium is quite degenerate because the economy will grow at a negative rate $-\kappa$. The root given by $r^G \frac{\psi - 1}{\psi}$ is also intuitive. When a firm invest $\psi$, it borrows $\psi - 1$ and thus will repay $r^G (\psi - 1)$ forever. The figure (1) suggests that one root is stable and one unstable. Fortunately, the stable root is the non degenerate one.

**Proposition 8** $b^*$ is an unstable root and $b^{**}$ is the stable root.

I now focus on the stable root and compute the corresponding investment and growth rate.

**Proposition 9** For the stable steady state, growth and investment rate given by the system of equations (17) are

\[
\begin{align*}
    i^{**} &= s\psi \pi - sp^G (\psi - 1) \\
    g^{**} &\simeq s\psi \pi - sp^G (\psi - 1) - \kappa
\end{align*}
\]  

(13a) \hspace{1cm} (13b)

An even simpler result may be obtained when I replace the leverage by its value with respect to $r^G, r^B$ and $\pi$ given by the incentive compatibility constraint

**Proposition 10** When replacing the leverage $\psi$ by its value given by equation (13)

\[
\begin{align*}
    i^{**} &= sr^B \\
    g^{**} &\simeq sr^B - \kappa
\end{align*}
\]

(14a) \hspace{1cm} (14b)

The investment rate is a function of the firm ”saving” rate $s$ and the interest rate charged on bad firm.

### 3.2 The response to a change in interest rate

**intuition** Equation (11C) shows that corporate investment is sensitive to the leverage $\psi$ and the debt level $b_t$. Both reacts to change in interest rate but in opposite direction and with a different timing.

The debt level $b_t$ is a state variable. A fall in $r$ will not affect $b_t$ on impact but will lower $r^G$ pushing downward $b_{t+1}$. The income of good firm will be unaffected in period $t$ but will increase in period $t + 1$ allowing an increase in retained earnings and in investment all things being equal. This income distribution channel is very intuitive and appears in every model with financial friction (see Bernanke Gertler 1989 and Kiyotaki and Moore 1995).

What is specific to the model is the response of the leverage $\psi$ As I have shown in the previous section, the leverage is an increasing function of the interest rate and will react instantaneously to a shock.
Permanent changes  A fall in real interest rate generates two opposite effects on investment. The response of steady state investment to a permanent fall of $r$ is ambiguous in first approximation. However, the equation (14A) shows that the leverage effect clearly dominates the income distribution effect. Investment is proportionnal to the interest rate on bad firms which is increasing with safe interest rate.

Figure (2) confirms this domination. It represents the steady state investment for various level of interest rate and shows an increasing, nearly linear function.

Temporary changes  I now explore the effects of a temporary change. I consider a fall of real rate from 4.5 percent to 2 percent for 5 periods. Then, $r$ go back to 4.5 percent. I plot the response in figure (3). On impact, leverage and corporate investment falls. They recover after five periods. After these five periods, interests repayments are lower than their steady state value. As a consequence, investment has a peak and is slightly higher than its steady state value. Then debt and investment converges to their long run values.

4 Literature

The paper is related to several strand of literature. It belongs in particular to adverse selection literature in macroeconomics and finance. Adverse selection is a popular friction in finance. According to Myers and Maljuf (1984), it may explain the pecking order of financing source for firms. It was also prominent in De Marzo and Duffie (1995) and Albuquerque and Hopenhayn (2004). It was less popular in macroeconomics compare to other frictions like costly state
verification (Townsend 1984) or Collateral constraint (Hart and Moore 1994). There are some exceptions, including the seminal paper of Stiglitz and Weiss (1981), but also papers by Kurlat (2013, forthcoming), or Boissay, Collard and Smets (2015). This lack of popularity is probably caused by the relative complexity of adverse selection problems. My first contribution is to provide a very simple model of adverse selection in infinite horizon. The second contribution is to show that adverse selection is a microfoundation for a linear relation between investment and cash flows. A significant correlation between the two is nearly systematic in investment equation regression. My model confirms that interest rate may have adverse effects under private information problems, an effect already found in Stiglitz and Weiss (1981). Boissay, Collard and Smets also find a reversed curve for loan demand in interbank market with adverse selection.

The paper is also related to other macroeconomic models with financial frictions (see Be- ranke and Gertler (1989), Kiyotaki and Moore (1995), Beranke, Gertler and Gilchrist (1999), Christiano, Motto, Rostagno (2014)). The key difference is that user cost of capital still have substantial effects on investment in these models.

Some papers have studied adverse effects of interest rate on output in macroeconomic model without investment. Bilbiie (2008) shows that Hand to Mouth consumers may reverse the sign of output euler equation. Cochrane (2015)sheows that lower rates may reduce output through the Fisher equation.

**Conclusion**

I aim to explain why fall in real interest rate may not be able to increase corporate investment. I provide a model of corporate investment with financial frictions which effectively eliminates any user cost of capital effect, the standard channel by which lower rates increase investment.
Like all economic models, it is based on questionable assumption but is simple and may be solved by hand, a non trivial feature for an endogenous growth model with adverse selection in infinite horizon. The main drawback of the model is that it is too successful in achieving its purpose. Not only it reduces positive effects of low interest rate on investment but it creates powerful negative ones. This negative effect is probably too large to fit macroeconomic datas. However, my model can be a good starting point to develop an alternative model of corporate investment. Several assumption can be relaxed like the exogeneity of the firm saving rate. Another possibility would be to make the incentive compatibility constraint not always binding.
References


A Proofs

A.1 Proof for proposition 1

Proof 1 It is obvious that firms do not want to borrow in order to invest, because return of investment $\pi$ is lower than repayments $r^B$.

I will now try to show that dividends will be set according to proposition 1. Suppose that bad firms set borrowing, dividends and investment following the proposition 1. Consider now a deviation for investment at period $0$. $d_0 = (\pi K_0 - B_0) - \delta d_0$ and $I_0 = \delta d_0$ Investment remains unchanged after period 0. Thus $\forall \ t > 0$, $d_t = (\pi K_0 - B_0) + \pi \delta d_0$. The deviation for the discounted sum of expected dividends is

$$\delta V_0 = -\delta d_0 + \pi \delta d_0 \frac{\lambda}{1 + r_0} Q_1^B = -\delta d_0 [1 - \frac{\pi}{r^B}] < 0$$

Deviating from the optimal strategy has a negative net present value

A.2 Proof for proposition 2

Proof 2 It is obvious that firms want to borrow as much as possible in order to invest, because return of investment $\pi$ is higher than repayments $r^G$.

I will now try to show that dividends will be set according to proposition 2. Suppose that good firms do not set dividends and investment following the proposition 2 and pay higher dividends. For example $\forall \ t$, $d_t = (1 - s')(\pi K_t^* - B_t^*)$ and $I_t^* = E^{max} + s'(\pi K_t^* - B_t^*)$ with $s' < s$. $(K_t^*)_{t \in \mathbb{N}^+}$ and $(B_t^*)_{t \in \mathbb{N}^+}$ are the implied sequence of capital stock and debt.

Consider now a deviation for investment at period 0. $d_0 = (1 - s')(\pi K_0 - B_0) - \delta d_0$ and $I_0 = \delta d_0$ Investment remains unchanged after period 0. Thus $\forall \ t > 0$, $d_t = (1 - s')(\pi K_t^* - B_t^*) + \pi \delta d_0$. The deviation for the discounted sum of expected dividends is

$$\delta V_0 = -\delta d_0 + \pi \delta d_0 \frac{\lambda}{1 + r_0} [(1 - \kappa)Q_1^G + \kappa Q_1^B] = -\delta d_0 [1 - \frac{\pi}{r^G}] > 0$$

Distributing less dividends and investing more has a positive net present value. If not constrained, firms want to lower dividends and increaseds investment

A.3 Proof for proposition 8

Proof 3 I denote $f$ the function $f(b_t) = \frac{b_t + r^G(\psi - 1)s(\pi - b_t)}{1 + s(\pi - b_t)}$

The derivative $f'(\pi) = 1 + s(\psi - r^G(\psi - 1))$. Because by assumption 1 $\pi > r^G$ it is clear that the derivative of $f$ is always superior to one when $b_t = b^*$. thus $b^*$ is an unstable root.

The derivative $f'(r^G \frac{\psi - 1}{\psi}) = \frac{1 + s(\psi - r^G(\psi - 1))}{1 + s(\psi - r^G(\psi - 1)) \psi} < 1$. So $b^{**}$ is a stable root.